SIMULATION OF MULTICONDUCTOR TRANSMISSION LINES BY PADÉ APPROXIMATION VIA THE LANCZOS PROCESS

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ABSTRACT

We introduce a mathematical model for lossy, multiconductor transmission lines that makes possible the use of the Padé approximation via the Lanczos (PVL) process to the analysis of complex linear networks that contain multiple transmission line systems. Results from numerical experiments are presented to demonstrate the validity and discuss the efficiency of the proposed model.

INTRODUCTION

With the rapid growth in size, density and complexity of modern integrated circuits, the use of classical circuit simulators becomes inefficient or even impossible. The alternative to such simulators is the development of approximate reduced order models which capture, with acceptable engineering accuracy, the important attributes of the response of the circuit over the bandwidth of interest to the specific analysis or design. For this purpose moment-matching techniques [1] have been applied to a variety of electrical CAD problems. Despite their spectacular success, moment-matching techniques are hindered by some numerical limitations [2][3].

Recently, a new method was introduced for the computation of the Padé approximation of a lumped linear RLC circuit via the Lanczos process [3]. This algorithm, which is called PVL (Padé via Lanczos), produces more accurate and higher-order approximations compared to asymptotic waveform evaluation (AWE) and its derivatives. Despite its superior performance to moment-matching techniques, applications of PVL have so far been limited to lumped RLC circuits. The objective of this paper is to present a method for modeling circuits with lossy multiconductor transmission lines using the PVL algorithm.

TRANSMISSION LINES

The general approach to include the multiconductor transmission line systems into a circuit simulator is to treat them as linear multiports described by a suitable relationship between terminal voltages and currents which is obtained rigorously from the Telegrapher's equations. For instance, the following formulation in Laplace domain is generally employed

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to integrate transmission lines into moment-matching type simulations

$$\mathbf{A}(s)\mathbf{V}_t(s) + \mathbf{B}(s)\mathbf{I}_t(s) = 0 \quad (1)$$

where $\mathbf{V}_t(s)$ and $\mathbf{I}_t(s)$ are column vectors containing, respectively, the terminal voltages and currents of the multiconductor line system. The matrices $\mathbf{A}(s)$ and $\mathbf{B}(s)$ are described in terms of the per-unit-length line parameters and are usually exponential type functions of s.

However, in order to be able to use the PVL technique to analyze circuits containing transmission lines, the elements of the matrices $\mathbf{A}(s)$ and $\mathbf{B}(s)$ in (3) need to be first-degree polynomials in s. For this purpose, we introduce a new mathematical model for lossy multiconductor transmission lines. The mathematical model is based on the use of Chebyshev polynomial expansions for the approximation of the spatial variation of the transmission-line voltages and currents. The choice of Chebyshev polynomials is motivated by the exponential rate of convergence of Chebyshev expansions [4]. This suggests that highly accurate approximations of the voltage and current distributions along the lines can be effected with a small number of polynomials (M). The choice of M will be discussed later.

In brief, the method used the development of the new model is as follows: First, the transmission-line voltages and currents are replaced by their Chebyshev expansions in the Telegrapher's equations. Then a simple collocation procedure is used to obtain a matrix representation of the transmission line equations with matrix coefficients that are first-degree polynomials in s, and in which terminal transmission-line voltages and currents appear explicitly

$$(\mathbf{A}^R + s\mathbf{A}^I)\mathbf{V}_t(s) + (\mathbf{B}^R + s\mathbf{B}^I)\begin{bmatrix} \mathbf{I}_t(s) \\ \hat{\mathbf{I}}(s) \\ \hat{\mathbf{V}}(s) \end{bmatrix} = \mathbf{0} \quad (2)$$

In (2) $\mathbf{A}^R$, $\mathbf{A}^I$, $\mathbf{B}^R$, and $\mathbf{B}^I$ are functions of line parameters, and $\hat{\mathbf{V}}$ and $\hat{\mathbf{I}}$, respectively, contain the voltages and currents at the collocation points except the near and far ends.

We enter (2) as a stamp into the overall circuit matrix. Therefore, it is important to keep the order of Chebyshev approximation as small as possible in order not to increase the size of the overall circuit matrix very much. The accuracy of the approximation, on the other hand, depends on the order of approximation. Using the exponential convergence rate property of the Chebyshev expansions, an optimum value for M, the number of collocation points, for a transmission line of length l can be chosen as,

$$M = 4\frac{l}{\lambda_{\min}} + 2, \quad (3)$$

where $\lambda_{\min}$ is the wavelength at the predetermined maximum frequency.

CIRCUIT FORMULATION

Consider a linear circuit $\mathcal{N}$ which contains linear lumped components and multiconductor transmission line systems. The time-domain modified nodal admittance (MNA) matrix equations for the circuit $\mathcal{N}$ with an impulse excitation as input can be written as

$$\mathbf{C}_{\mathcal{N}}\frac{d\mathbf{v}_{\mathcal{N}}(t)}{dt} + \mathbf{G}_{\mathcal{N}}\mathbf{v}_{\mathcal{N}}(t) + \sum_{k=1}^{K}\mathbf{P}_k\,\mathbf{i}_k(t) = \mathbf{b}_{\mathcal{N}}\delta(t) \quad (4)$$

where $\mathbf{v}_{\mathcal{N}}(t)$ is a vector of size $N_{\mathcal{N}}$ containing the waveforms of the node voltages, voltage source and inductor currents; $\mathbf{b}_{\mathcal{N}}\delta(t)$ is a vector representing the excitations; $\mathbf{G}_{\mathcal{N}}$ and $\mathbf{C}_{\mathcal{N}}$

are constant matrices; $\mathbf{P}_k$ is a $N_{\mathcal{N}} \times 2n_k$ selector matrix whose entries are 1 or 0, that maps $\mathbf{i}_k(t)$, the terminal currents of the kth line system, into the node space of the circuit $\mathcal{N}$; K is the number of transmission-line systems and n_k is the number of conductors in the kth line system.

The transmission lines are described in the frequency domain by (2). Combining the formulations for all transmission lines with the Laplace transform of (4), one obtains the frequency-domain MNA matrix for the circuit $\mathcal{N}$

$$(\bar{\mathbf{G}} + s\bar{\mathbf{C}})\mathbf{X} = \mathbf{b} \tag{5}$$

Let $H(s)$ be the output of interest,

$$H(s) = \mathbf{d}^T\mathbf{X}(s). \tag{6}$$

Then, using (5), the output frequency response is given by

$$H(s) = \mathbf{d}^T(\bar{\mathbf{G}} + s\bar{\mathbf{C}})^{-1}\mathbf{b} \tag{7}$$

The PVL algorithm, now, can be applied to (7) to find the Padé approximation of the frequency response via Lanczos process,

$$H_q = \sum_{i=1}^{q} \frac{k_i}{s - p_i} \tag{8}$$

where q is the order of approximation, and p_i and k_i are, respectively, the poles and the corresponding residues. The PVL algorithm can be found in [3].

EXAMPLES

The first example deals with the interconnection circuit shown in Fig. 1. This example has been widely used in the experimental validation of moment-matching techniques [2]. We applied the PVL algorithm to this circuit and the 35th order PVL approximation

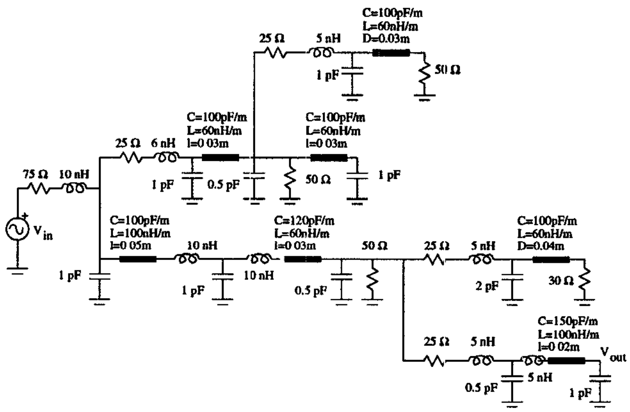

Figure 1: The interconnect circuit.

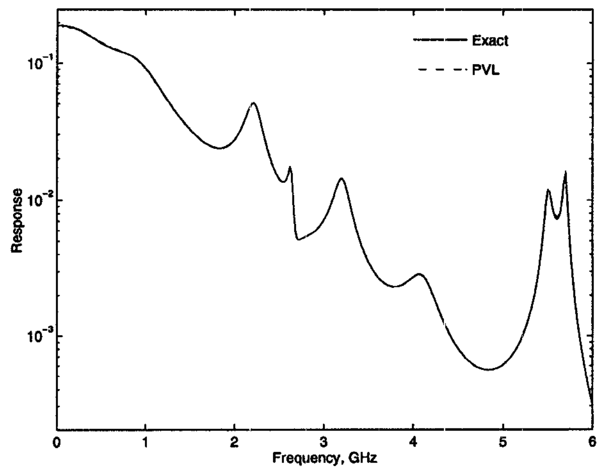

Figure 2: The output frequency response of the interconnect circuit.

is compared with the exact response for V_{out} in Fig. 2. The two responses are indistinguishable. The same circuit was also analyzed using the multipoint moment-matching technique [5]. To find the response from dc to 5 GHz, four expansions were needed for the moment matching case, which means that the matrix had to be factored 4 times (1 real, 3 complex). On the other hand, using PVL the same response is extracted from only one real (but larger) matrix factorization. Moreover, the PVL technique does not involve the additional cost of calculating the moments of the

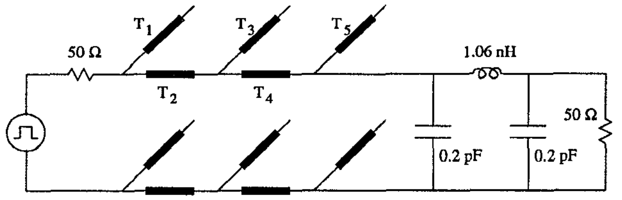

Figure 3: The low-pass filter circuit.

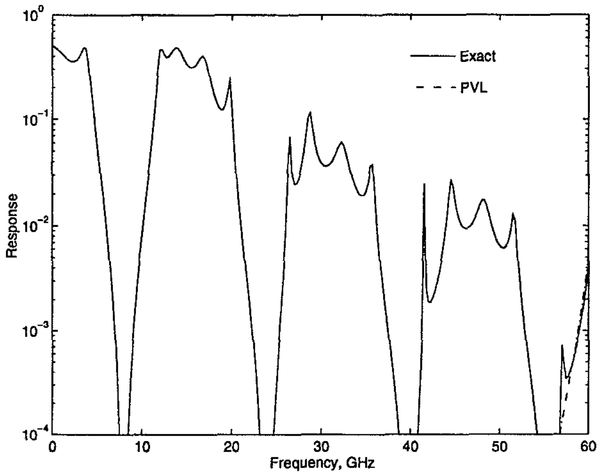

Figure 4: The output frequency response of the low-pass filter circuit.

transmission lines.

The second example considers a low-pass filter implemented with transmission lines as shown in Fig. 3. The filter has a cut-off frequency of 4 GHz. All transmission-line sections are assumed lossless and are of length $\lambda/8$ at the cut-off frequency. Their per-unit-length parameters are: $L_1 = L_5 = 2.1633$ nH/cm, $C_1 = C_5 = 0.5136$ pF/cm; $L_2 = L_4 = 7.25$ nH/cm, $C_2 = C_4 = 0.15326$ pF/cm; $L_3 = 2.3433$ nH/cm, $C_3 = 0.47416$ pF/cm. This filter was successfully analyzed in [5] using multipoint Padé approximation with expansion points at 0, 12.5, 25, 37.5, and 50 GHz. The response obtained after 70 PVL iterations is compared with the exact response in Fig. 4. The agreement over the 0 GHz - 50 GHz band is excellent.

CONCLUSIONS

We have introduced a mathematical model that allows circuits with multiple lossy multiconductor transmission lines to be simulated efficiently and accurately by the PVL algorithm. The mathematical model is based on the use of Chebyshev expansions for the representation of the spatial variation of the transmission-line voltages and currents.

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